Chapter 9

Right Triangles and Trigonometry

Section 4
Special Right Triangles

GOAL 1: Side Lengths of Special Right Triangles

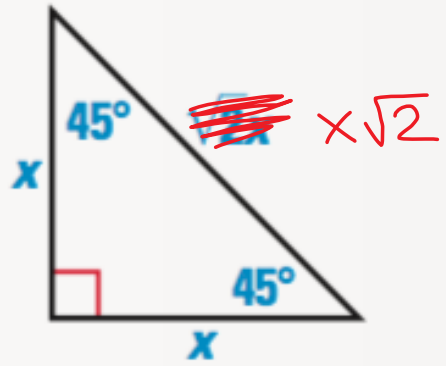
Right triangles whose angle measures are 45°-45°-90° or 30°-60°-90° are called **special right triangles.** In the Activity on page 550, you may have noticed certain relationships among the side lengths of each of these special right triangles. The theorems below describe these relationships. Exercises 35 and 36 ask you to prove the theorems.

THEOREMS ABOUT SPECIAL RIGHT TRIANGLES

THEOREM 9.8 *45°-45°-90° Triangle Theorem*

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

45 45 90
x x x√2

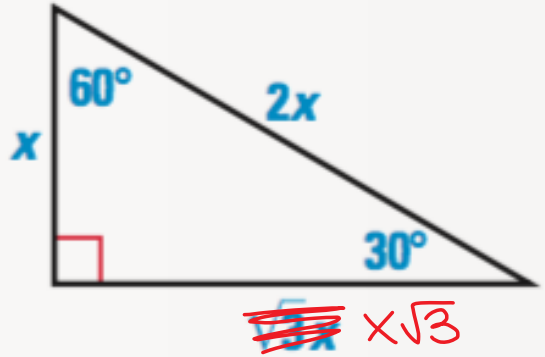


Hypotenuse = $\sqrt{2}$ · leg

THEOREM 9.9 *30°-60°-90° Triangle Theorem*

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

30 60 90
x x√3 2x



Hypotenuse = 2 · shorter leg
Longer leg = $\sqrt{3}$ · shorter leg

Example 1: Finding the Hypotenuse in a 45-45-90 Triangle

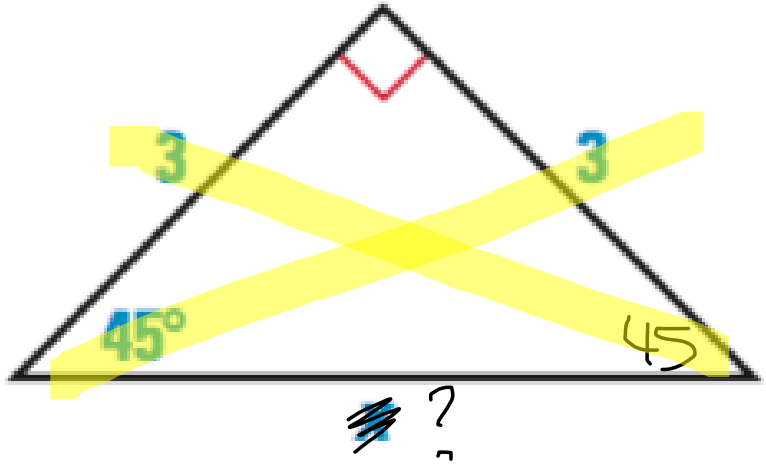
Find the value of x.

45 45 90

x x x√2

3 3 3√2

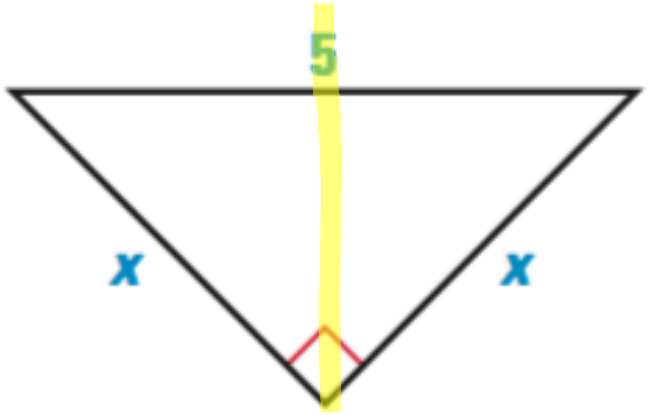
x = 3



Given the leg value → to find hypotenuse → "tack on" √2

Example 2: Finding a Leg in a 45-45-90 Triangle

Find the value of x.



45 45 90

x x $x\sqrt{2}$ 5

$\frac{5\sqrt{2}}{2}$ $\frac{5\sqrt{2}}{2}$

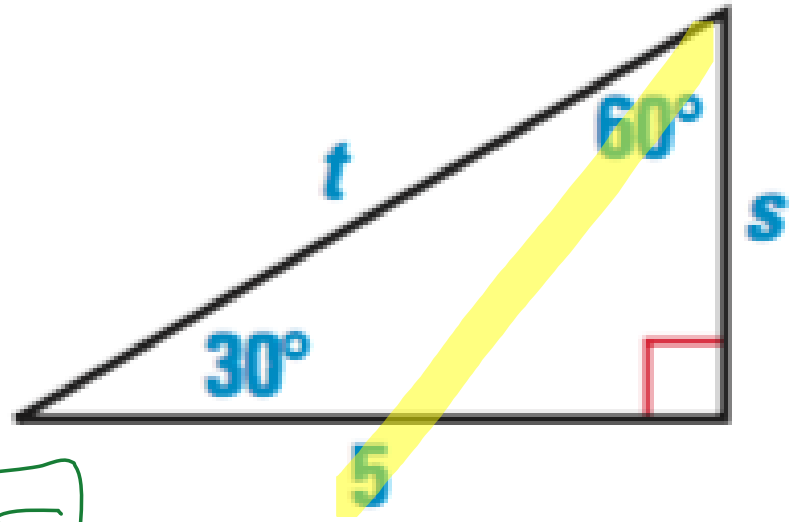
$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

$x = \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \rightarrow x = \frac{5\sqrt{2}}{2}$

When hyp. Is a whole # → to find leg → "tack on" $\frac{\#\sqrt{2}}{2}$

Example 3: Side Lengths in a 30-60-90 Triangle

Find the values of s and t.



30

x

$\frac{5\sqrt{3}}{3}$

60

$x\sqrt{3}$

5

90

$2x$

$\frac{2}{1}\left(\frac{5\sqrt{3}}{3}\right) = \frac{10\sqrt{3}}{3}$

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{5}{\sqrt{3}}$

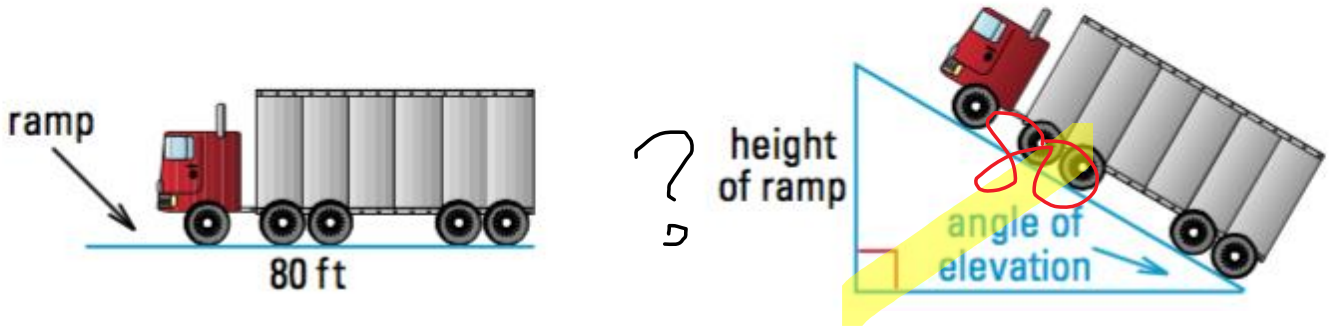
$x = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \rightarrow x = \frac{5\sqrt{3}}{3}$

When longer leg is a whole # → to find x → tack on $\frac{\#\sqrt{3}}{3}$

GOAL 2: Using Special Right Triangles in Real Life

Example 4: Finding the Height of a Ramp

TIPPING PLATFORM A tipping platform is a ramp used to unload trucks, as shown on page 551. How high is the end of an 80 foot ramp when it is tipped by a 30° angle? by a 45° angle?



30 — X — 40

60 — $x\sqrt{3}$

90 — 2x — 80

$2x = 80$

$x = 40$

45 — X — $40\sqrt{2}$

45 — X

90 — $x\sqrt{2}$ — 80

$x\sqrt{2} = 80$

$x = \frac{80\sqrt{2}}{2} \rightarrow 40\sqrt{2}$

Example 5: Finding the Area of a Sign

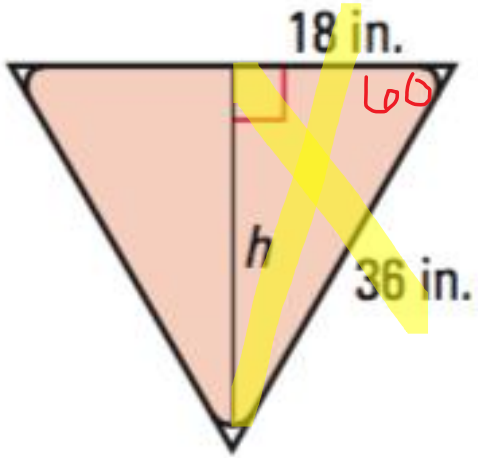
ROAD SIGN The road sign is shaped like an equilateral triangle. Estimate the area of the sign by finding the area of the equilateral triangle.

① 30 60 90

X $X\sqrt{3}$ 2X

18 $18\sqrt{3}$ 36

$X = 18$



② $A = \frac{1}{2}bh$

$\frac{1}{2}(36)(18\sqrt{3})$

$561.2\ in^2$

EXIT SLIP

45 45 90

→Given leg → that is x → put in the ratio for other x's

→Given hyp. (whole #) → to find x → $\frac{\#\sqrt{2}}{2}$

→Given hyp. ($\#\sqrt{2}$) → # in front is x

30 60 90

→given shortest leg → that is x → put in the ratio for other x's

→Given longer leg (whole #) → to find x → $\frac{\#\sqrt{3}}{3}$

→Given longer leg ($\#\sqrt{3}$) → # in front is x

→Given hyp. (whole #) → divide by 2 to find x